Comprehensive Mathematical Model for Freezing Time Prediction of Finite Object

Andrea Bassani[1,*], Guillermo D. Garrido[1], Gianluca Giuberti[1], Roberta Dordoni[1], Giorgia Spigno[1]
[1]Università Cattolica del Sacro Cuore, Department for Sustainable Food Process (DiSTAS), Via Emilia Parmense 84, 29122 Piacenza, Italy;
andrea.bassani@unicatt.it

UNIVERSITÀ CATTOLICA del Sacro Cuore

INTRODUCTION

COACh – Cold management in Agro-food Chains: solutions for process digitalization

Main goal

Develop a model to simulate the refrigeration process for energy optimization

1) *Empirical Models*

$$t = \frac{d_c}{E_f h}\left[\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right]\left(1 + \frac{N_{Bi}}{2}\right)$$

- 👍 Computational effort
- 👎 Only freezing time

2) *CFD or Theoretical Models* → solve heat transfer equation in 3 dimensions

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)$$

- 👍 Accurate
- 👍 No empirical formula
- 👎 Computational effort

3) Mixed → heat transfer equation solved in one dimension using the "method of lines"

$$\rho(T) C_p(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right]$$

- 👍 ***T variation vs time***
- 👎 Empirical formula
- 👍 **Computational effort**

FOOD PROPERTIES

1. Protein
2. Fat
3. Carbohydrate
4. Fiber
5. Ash
6. Water

- the thermal properties of the food as a function of temperature
- ***Flexible*** → can represent different kind of food
- The ***analytical methods*** widely used and validated
- Adapt the freezing times according to the **different batches**

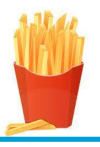
REFRIGERATION PROPERTIES

- ***Initial freezing point*** $T_f = -4.66\left(\frac{w_{carb}}{w_w}\right) - 46.4\left(\frac{w_{ash}}{w_w}\right)$
- ***Water bounded*** $w_b = 0.45 w_{prot} + 0.3 w_{carb}$
- ***Ice Fraction*** $w_{ice} = (w_{w0} - w_b)\left(1 - \frac{T_f}{T}\right)$
- ***Convective heat transfer coefficient*** $Nu = 0.664 (Re)^{0.5} (Pr)^{1/3}$ if $Re < 10^5$ else $Nu = 0.664 (Re_c)^{0.5} (Pr)^{1/3} + 0.037 (Re)^{0.8} (Pr)^{\frac{1}{3}}\left(1 - \left(\frac{Re_c}{Re}\right)^{0.8}\right)$

PROBLEM & SOLUTION

Shortcoming of mixed model

Impossibility to evaluate the temperature variation for food treated with finite shape, even if simple, such as a parallelepiped or a cylinder (e.g grilled eggplants or French fries).



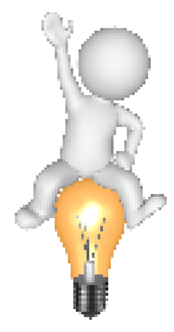
SOLUTION

Model proposed by Ferreira and Rojas (2019)

$$\rho(T) C_p(T) \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right]$$

+

EHTD (Equivalent Heat Transfer Dimensionality) shape factor approach E_f



$$\rho c_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left[k(T)\frac{\partial T}{\partial x}\right] E_f$$

Boundary conditions

$$\begin{cases} T = T_0 & -L \le x \le +L \quad t = 0 \\ -k(T)\frac{\partial T}{\partial x} = 0 & x = 0 \quad t \ge 0 \\ -k(T)\frac{\partial T}{\partial x} = h(T - T_a) & x = L \quad t \ge 0 \end{cases}$$

- The resulting partial differential equation was ***solved using the Method of Lines*** (MOL)
- discretization of the spatial derivative
- The resulting ordinary differential equation system (ODE) is solved using **Matlab 2021a**

RESULTS & DISCUSSIONS

Model Validation

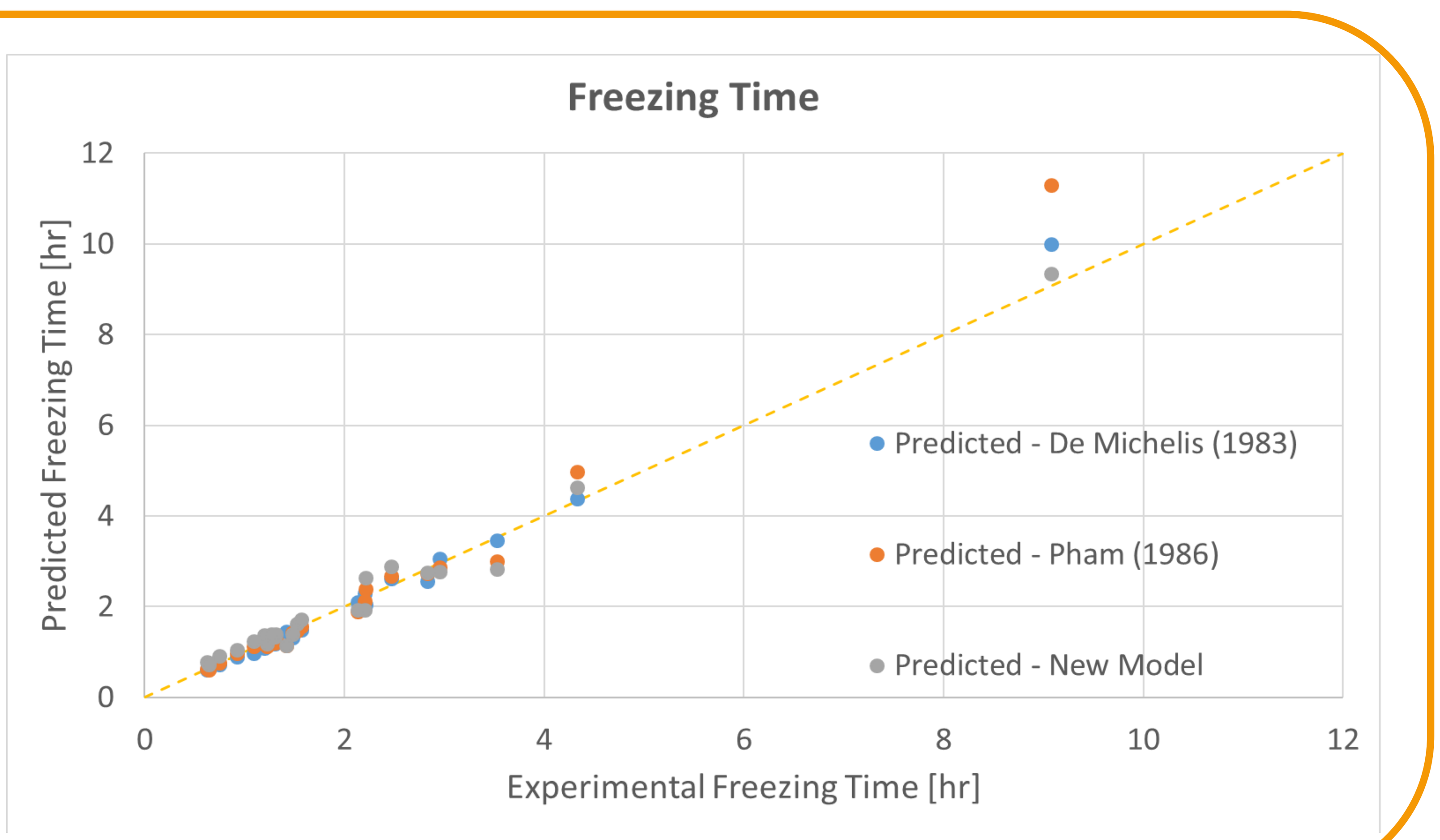
- Different freezing times in blocks of beef frozen in molds

the initial composition of the beef has been assumed

	Beef – De Michelis and Cavelo (1983)
Water	0.6340
Protein	0.3064
Fat	0.0528
Carbohydrate	0.0015
Fiber	0.0000
Ash	0.0053

- ***Good agreement*** with experimental data and with the other models
- The ***slight differences*** → assumption of the food composition

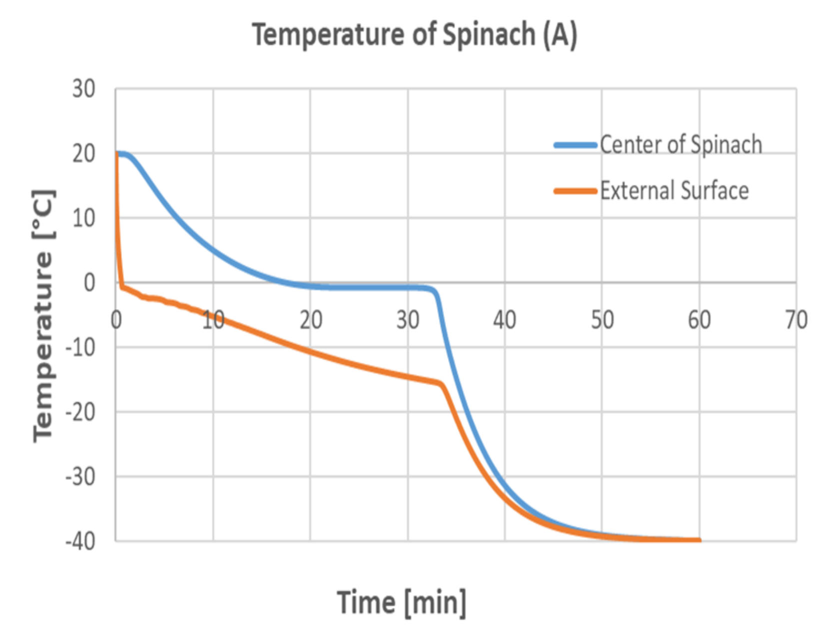
	R^2
De Michelis e Calvelo (1983)	0.9953
Pham (1986)	0.9821
New Model	0.9816

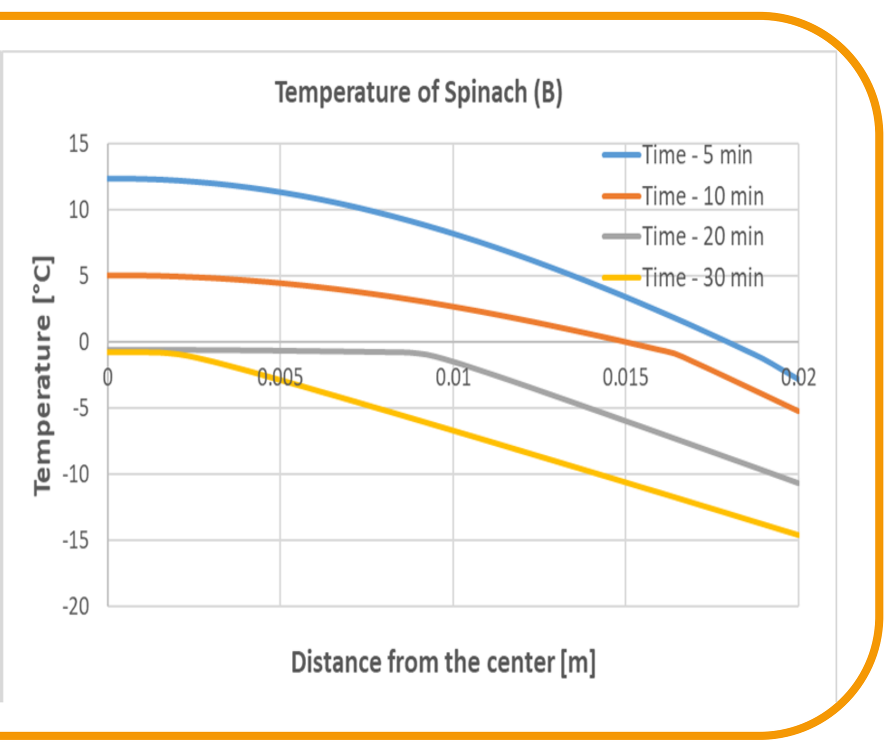


Model Simulation

- The principal dimension of the spinach is 0.04 m.
- The operating conditions of the refrigeration process have been hypothesized
  - the air temperature: - 40°C
  - the initial temperature of the spinach: 20°C

	Spinach
Water	0.9010
Protein	0.0350
Fat	0.0070
Carbohydrate	0.0290
Fiber	0.0190
Ash	0.0090





CONCLUSIONS & FUTURE DEVELOPMENTS

- **Development and validation** of a mathematical model for the **freezing** process.
- The developed model **is efficient, flexible and accurate** if the considered food can be assimilated to a defined geometric figure
- This model can be used to **reduce the freezing time of food** and to **energetically optimize** the process or the production of the cooling fluid.
- This model cannot give the same amount of information compared to a CFD simulation. → a CFD simulation at optimal process conditions.

References

- Ferreira S. R., Rojas L. O. A, 2019, Freezing times using time derivative of temperature on surface of foods, International Journal of Refrigeration, 98, 436-443.
- De Michelis A., Calvelo, A., 1983, Freezing time predictions for brick and cylindrical-shaped foods, Journal of Food Science, 48(3), 909-913
- Pham Q. T., 1986, Simplified equation for predicting the freezing time of foodstuffs, International Journal of Food Science & Technology, 21(2), 209-219.

Acknowledgments

COACh

The Project is co-financed by the Regional Operational Programme of the European Regional Development Fund (ERDF ROP 2014-2020) and by the Development and Cohesion Fund of Emilia-Romagna Region